Redefinition of the SI for the 21st Century

Murray Early
Electrical Metrology

World Metrology Day, 22 May 2018

Acknowledgements: Greta (Library), Vladimir and rest of Electrical Standards Team, and MSL colleagues

Measurement Standards Laboratory of New Zealand

A business of Callaghan Innovation

Redefinition of the SI…..Overview

- A bit of SI history
- The need for change to the SI
- Preparations for change
- Consequences for users
- The challenge of electrical units

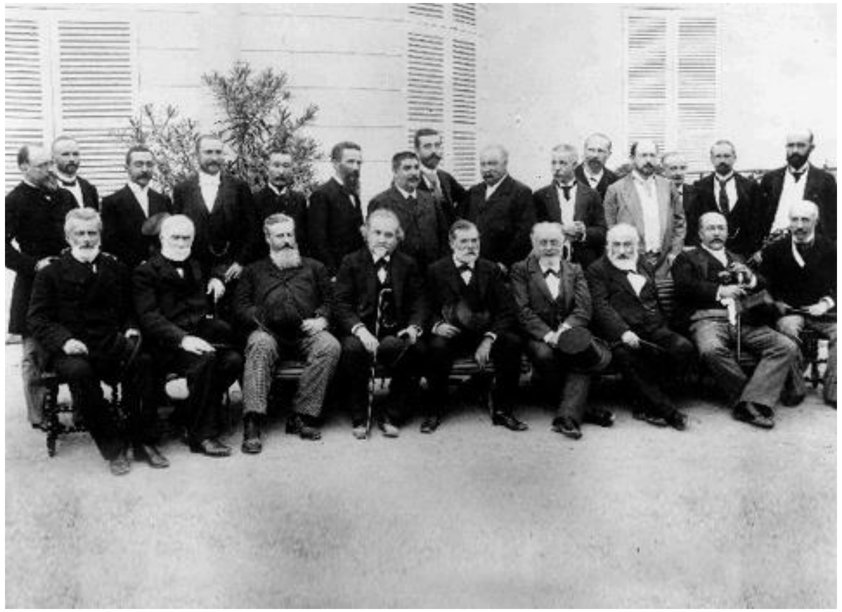
French: Système International d'Unités (hence 'the SI')

English: The International System of Units

Metrology: the science of measurement
(…or the science of not being wrong)

Another Revolution is Coming….

- 20th May 1875: the signing of the “Convention du Mètre”



2nd CGPM 1895 (www.bipm.fr)

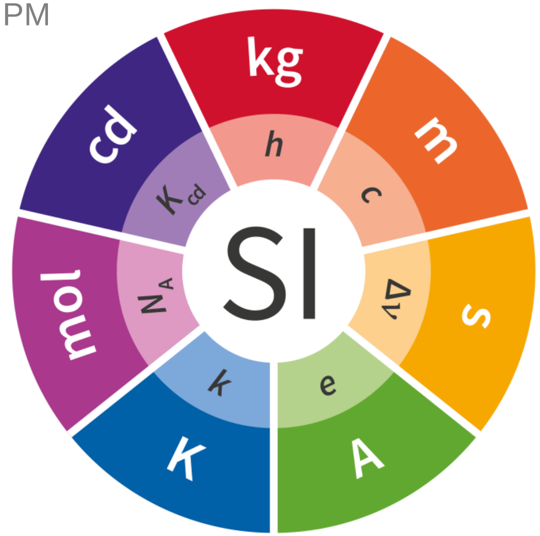
Art: 14

La présente Convention sera ratifiée suivant les Lois constitutionnelles particulières à chaque État; les ratifications en seront échangées à Paris dans le délai de six mois ou plus tôt, si faire se peut. Elle sera mise à exécution à partir du 1er Janvier 1876.

En foi de quoi, les Plénipotentiaires respectifs l'ont signée et y ont apposé le cachet de leurs armes.

Fait à Paris, le 20 Mai 1875.

Image: BIPM



- 20th May 2019: (144th anniversary) implementation of an extensive overhaul of the SI

The Metre Convention

- 20th May 1875: 17 countries signed the Metre Convention….
  - signed by the USA, but Britain (1884), Australia (1947)
  - New Zealand signed when? 1991
- 80 years prior to 1875 the idea of common measures caught up in the utopian ideals of the French Revolution (1789-1799)

"A tous les temp. A tous les peoples."
(For all time. For all people.)
Condorcet, Académie des Sciences, ~1792

- Turned to the Académie des Sciences: Coulomb, Lavoisier, Lagrange, Laplace, Cassini…

"The Metric system grew directly from the most radical period of the French Revolution; scientific revolutions often go hand-in-hand with political upheaval"
J H Williams, Defining and Measuring Nature, 2014

Standardisation of Units

- Multiplicity of units a major problem (hundreds of local units)

Braunschweig Market Length Standard (the Elle): 57.07 cm
"The required length measure for use at the market at least since the 16th century"

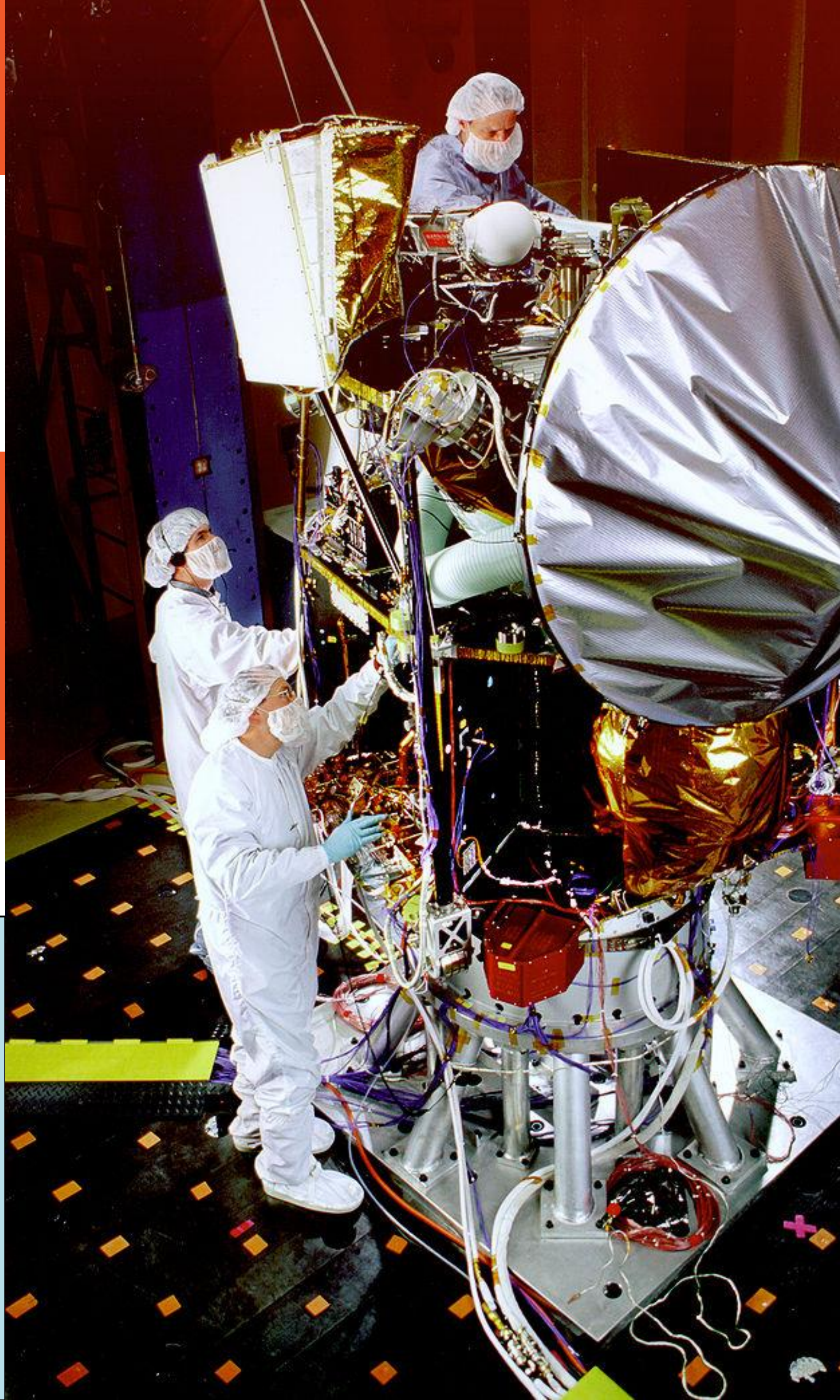
(by 1810 there were 112 different standards for the Elle around Germany)

- Initially covered length, mass and time (m, kg, s)

<u>CGS vs SI</u>

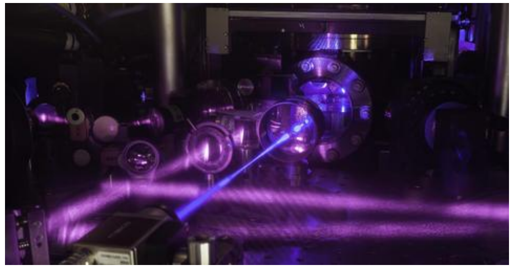
- Base units – same in practice (e.g. both have 1 g and 1 kg)
- Derived units – different names, vary by powers of 10

e.g. (CGS force) 1 dyne = 1 g x 1 cm/s^2 = 10^{-3} kg x 10^{-2} m/s^2 = 10^{-5} N (SI force)



Unit Systems

- Purpose – compromise to ensure common global language/understanding:
  - Trade – critical
    - NB: could once tax by using different units for buying and selling
    - niche units still widespread (e.g. diamonds by 1 carat = 0.2 g)
  - Science – simplicity if stay in the SI (SI units, SI equations)
    - niche units suit certain disciplines – e.g. natural units h=c=1
  - Public good – fair, trustworthy and coherent society
    - regulations, speed, medicine, safety etc
- Unit systems are science-led/informed but not scientifically derived
    - quite a lot of freedom how we express quantities (and units and formulae)
- Choose unit scales to suit us (prefer smallish numbers > 1)
    - e.g. SI base units: 1 m, 1 s, 1 kg, change of 1 K, 1 mol, 1 Cd, 1 A



Development of the Metric System

- 1790: meter defined as one ten-millionth of the distance between the North Pole and the Equator
- 1875: metre and kilogram artefacts (Platinum Iridium)
- 1921: all physical quantities (including ampere)
- 1949: ampere becomes 4th base unit (Giorgi)
- 1960: kelvin and candela added to base units
  - now called the “Système international d'unites” (the SI)
  - meter defined as 1 650 763.73 wavelengths of the orange-red line of the krypton-86 atom
- 1971: mole becomes 7th base unit
- 1983: speed of light defined – the metre is based on a fundamental constant
- 1990: conventional values for electrical units (voltage and resistance) ~ non-SI…

The trend in the International System of Units (SI):

physical artefacts "properties of our planet" - not invarient → atomic parameters invariant, empirical rather than fundamental → quantum constants invariant and fundamental

Report on the Calibration of a 100 Ω resistor by the BIPM
March 2017

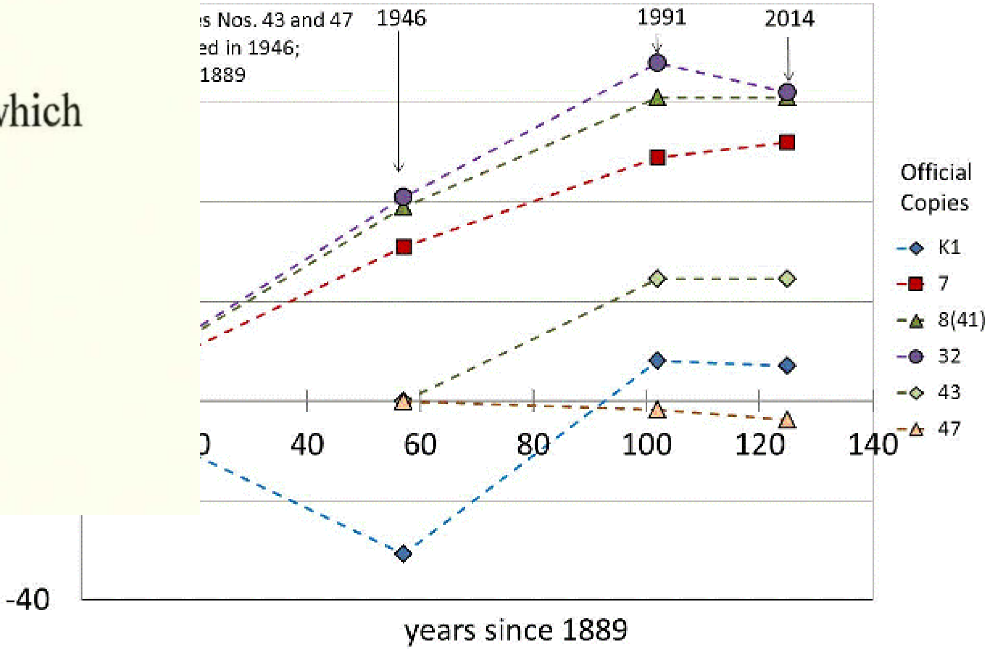
u is calculated by addition in quadrature of:

- The relative standard uncertainty u_1 determined from the relative standard deviation of the mean (on the mean date) of the series of measurements. It is calculated after removal of any possible short-term drift: $u_1 = 2.0 \times 10^{-9}$

- The relative standard uncertainty u_2 arising from the combined contributions associated with the BIPM measurement facility and the traceability: $u_2 = 18 \times 10^{-9}$

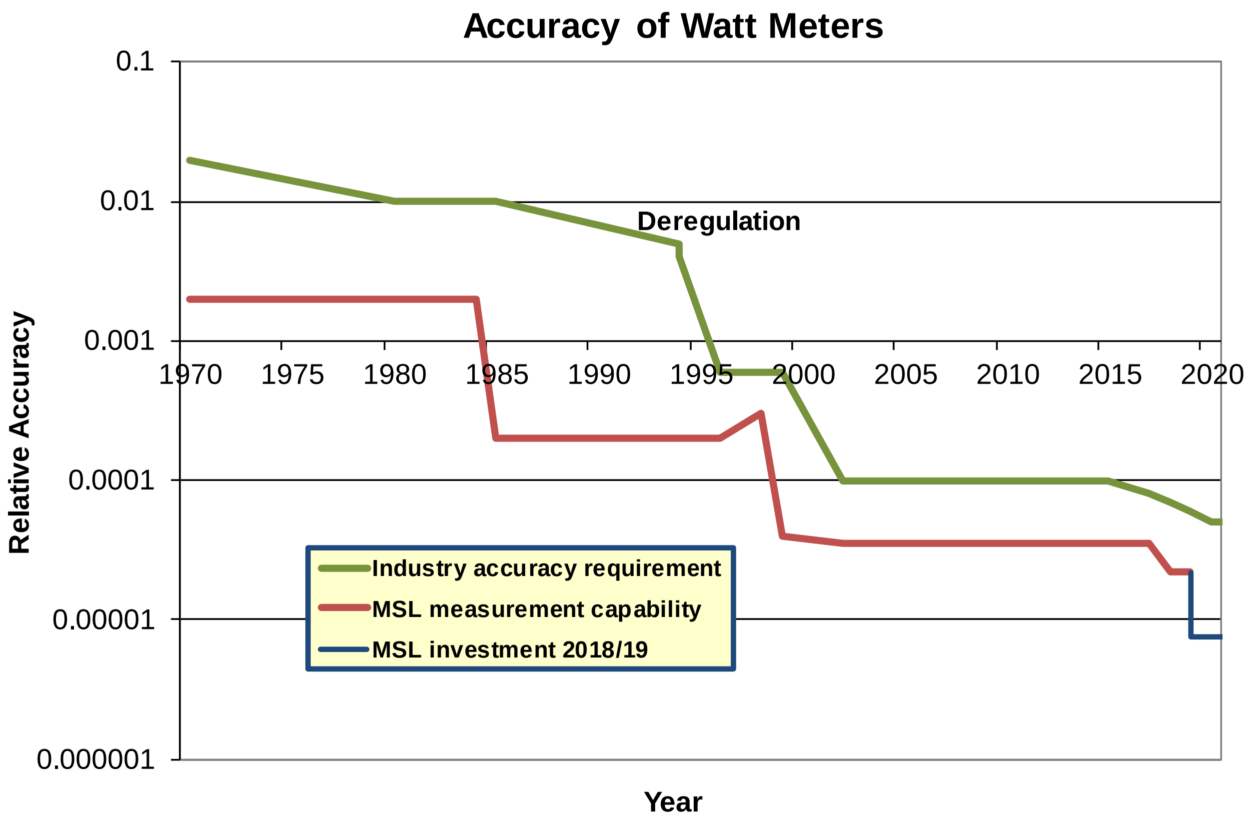
All the uncertainties given in this certificate are estimated standard uncertainties, without the application of a coverage factor k.

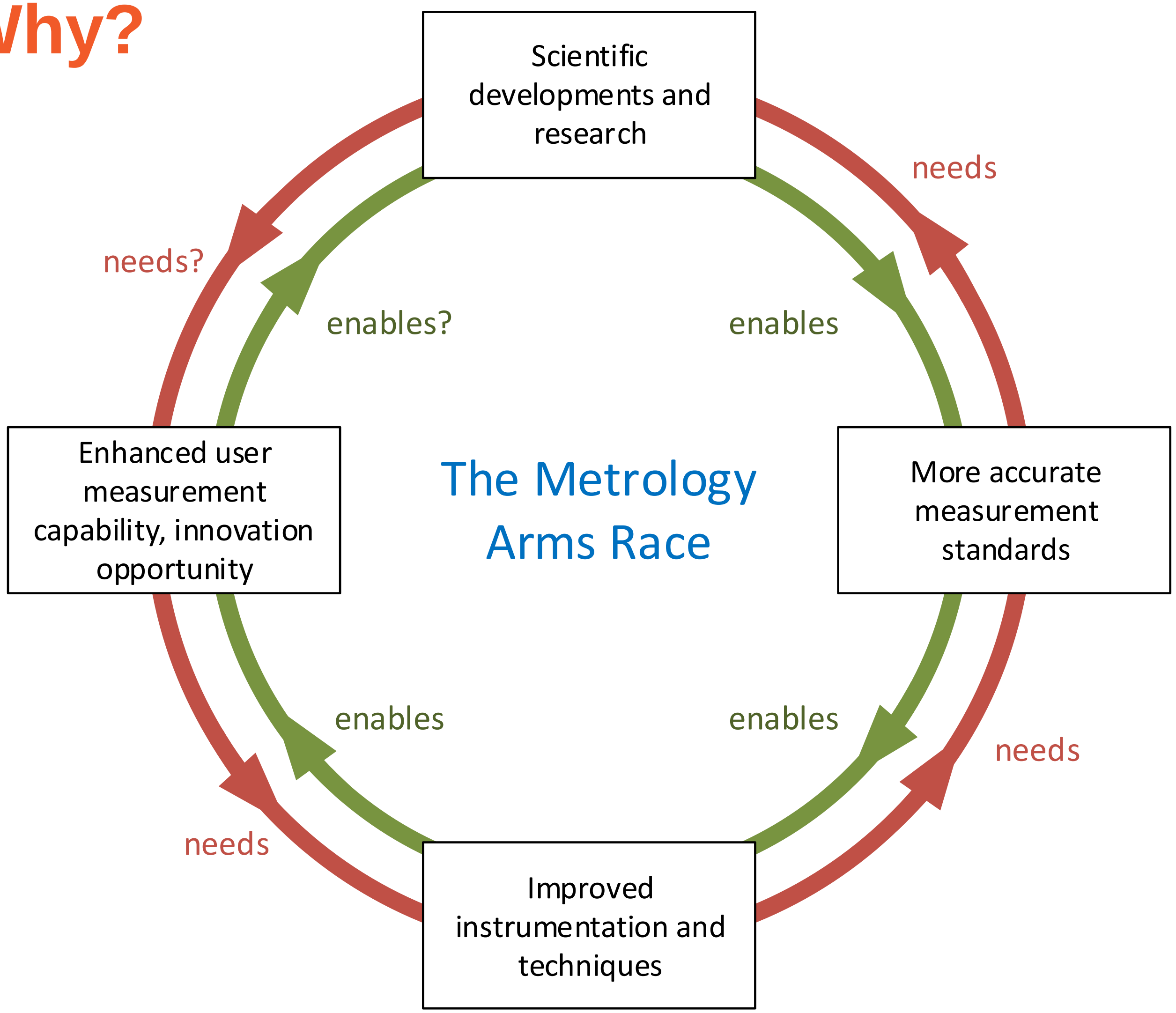
The standard uncertainty associated with the use of the recommended value of $R_{K\text{-}90}$, which has a relative value of 1×10^{-7}, has not been included.



Aside: Improving Accuracy – Why?

1. Metrological 'arms race'



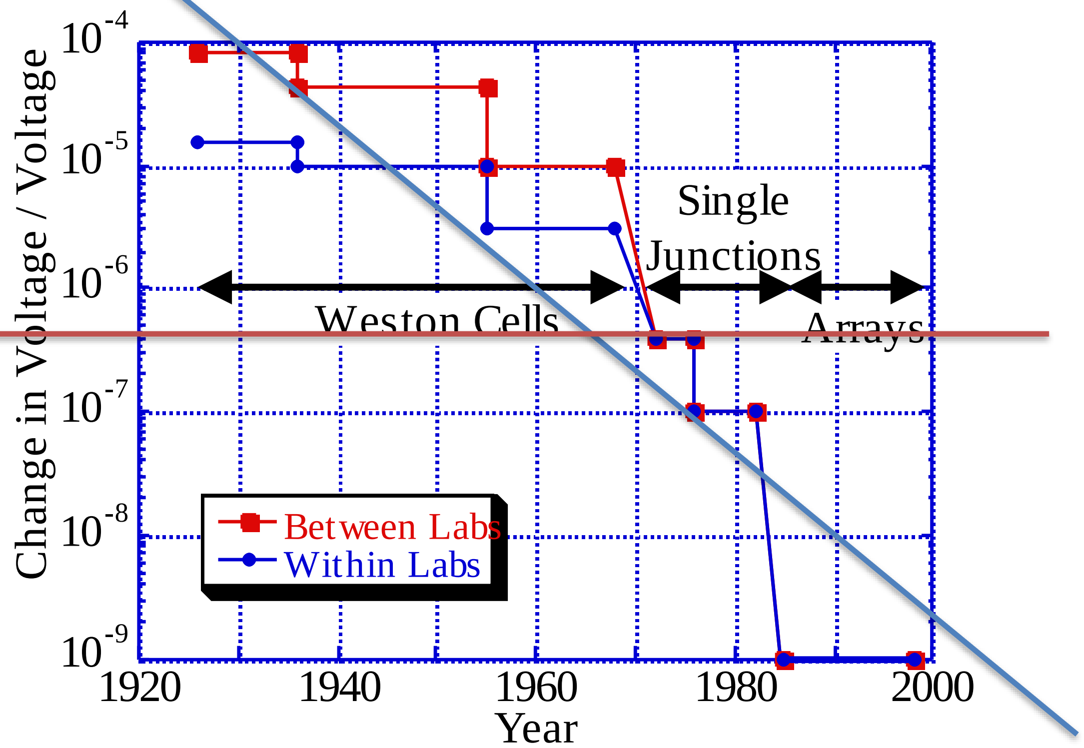


Aside: Improving Accuracy – Why?

2. Metrological Moore's Law: $\frac{\Delta u}{u} = 10^{-\Delta t/15\ years}$
   (improve accuracy by about a factor of 10 every 15 years)

Example: dc voltage

K_{J-90} uncertainty (conventional value of 2e/h)



From Bachmair, 1988 and Hamilton, 199ε

8.5 digit Digital Multimeters

$$\frac{\Delta u}{u} = 10^{-\Delta t/15\ years}$$

Measurement Standards Laboratory of New Zealand

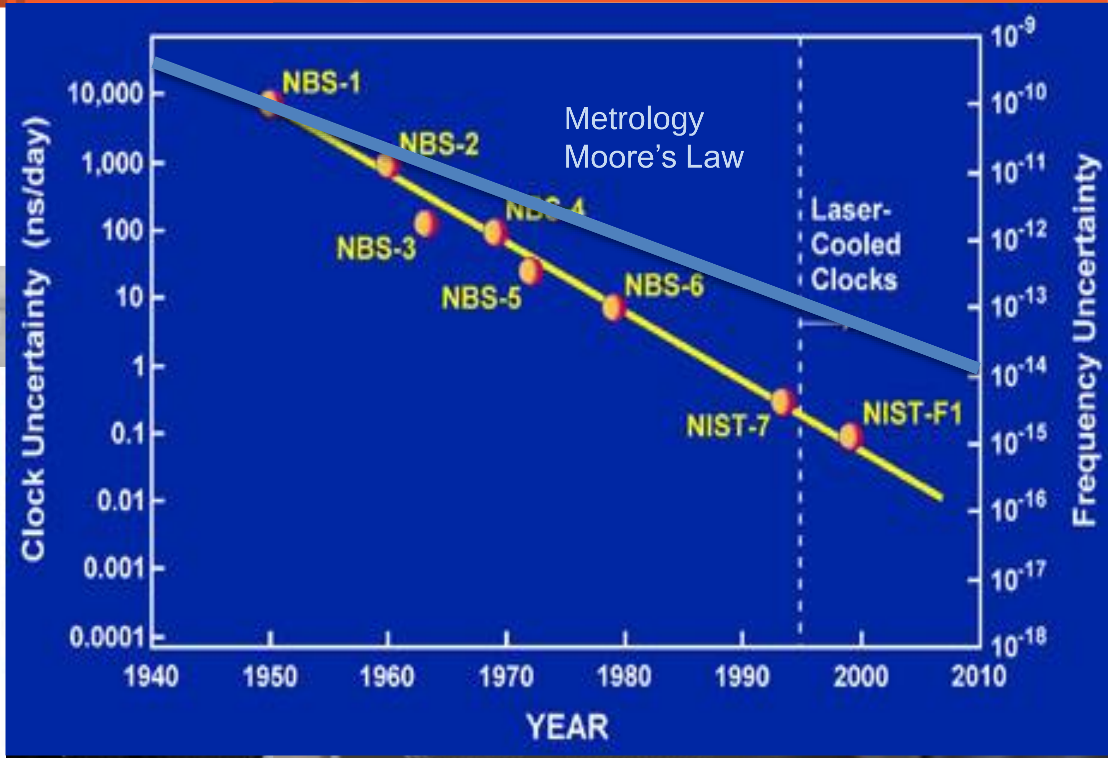
Aside: Improving Accuracy – Why?

3. Basis for genuine innovation

Value of GPS – 2013 US Economic Benefits

	Application Category	Range of Benefits ($billions)	Mid-range Benefits ($billions)
A	Precision Agriculture — grain*	10.0–17.7	13.7
A	Construction — earthmoving with machine guidance*	2.2–7.7	5.0
A	Surveying	9.8–13.4	11.6
A	Air Transportation	.119–.168	0.1
C	Rail Transportation — positive train control	.010–.100	0.1
C	Maritime Transportation — private-sector use of nautical charts and related marine information*	.106–.263	0.2
A	Fleet vehicle connected telematics*	7.6–16.3	11.9
A	Timing —average of eLoran and GEOs estimates	.025–.050	0.1
A, B	Consumer and Other Non-Fleet Vehicle — average of estimates based on willingness-to-pay and value of time*	7.3–18.9	13.1
	TOTAL	37.1–74.5	55.8

A = confident, B = indicative, C = notional.
*Includes benefits from purchase input cost savings.
Note: Numbers may not add to totals due to rounding.



Precision Measurement…

…brings the world into focus

Concept: Michael de Podesta, NPL

Photo: Hamish Edgar, MSL

Electrical Units – Accuracy Drivers

- Electrical measurements revolutionised by quantum effects:

Year		Equation		Constant
1962	Josephson effect	$V = n\left(\frac{h}{2e}\right)f$	Josephson constant	$K_J = \frac{2e}{h}$
1980	Quantum Hall effect	$R = \frac{1}{i}\left(\frac{h}{e^2}\right)$	von Klitzing constant	$R_K = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha}$
1987	Single electron counting	$I = nef$	Electronic charge	e

NB: $\alpha \cong 1/137$, is the fine structure constant

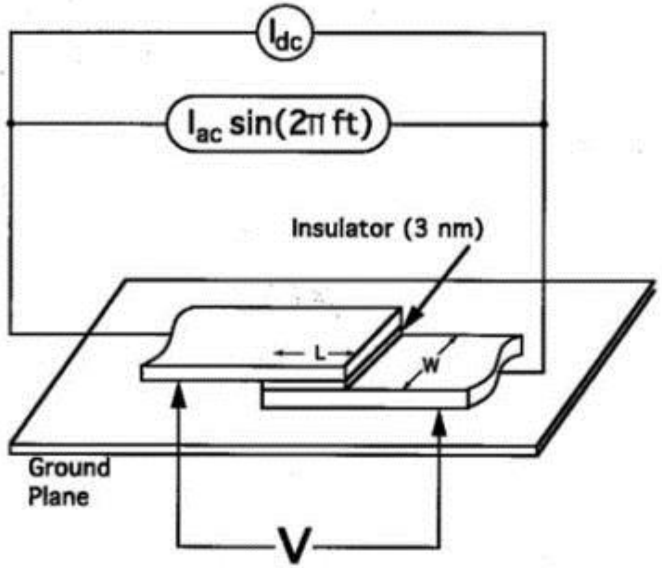
- 1990 - Conventional values defined: [NB: could redefinition have occurred then?]
  - K_{J-90} = 483 597.9 GHz/V (u_r = 0.4 ppm)
  - R_{K-90} = 25 812.807 Ω (u_r = 0.1 ppm)
  - uncertainties dominated by link to mechanical units (force, energy, power) reproducibility > 1000 times better!
  - best calibrations beyond the SI

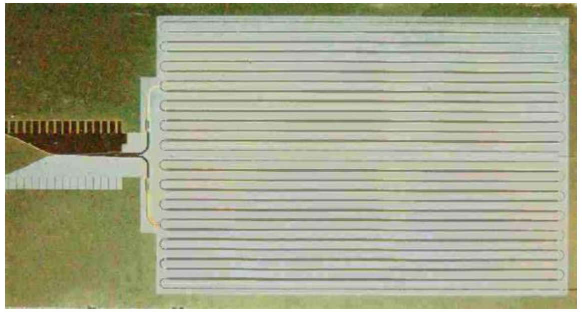
The Josephson Effect

- Josephson **SIS** junction:
  - **S**uperconductor-**I**nsulator-**S**uperconductor sandwich
  - radiated by microwaves, frequency f
  - small dc voltage generated
- 1990s key advance: put 10^4 to > 10^5 in a series array – 10 V output!
  - Mature technology: > 70 arrays in use world wide (NMI, industry and the military)
  - Various kinds of junctions (SNS, SINIS,…)
  - Various array designs (conventional, programmable, pulse)
  - Programmable AC voltages – applied to impedance and power standards

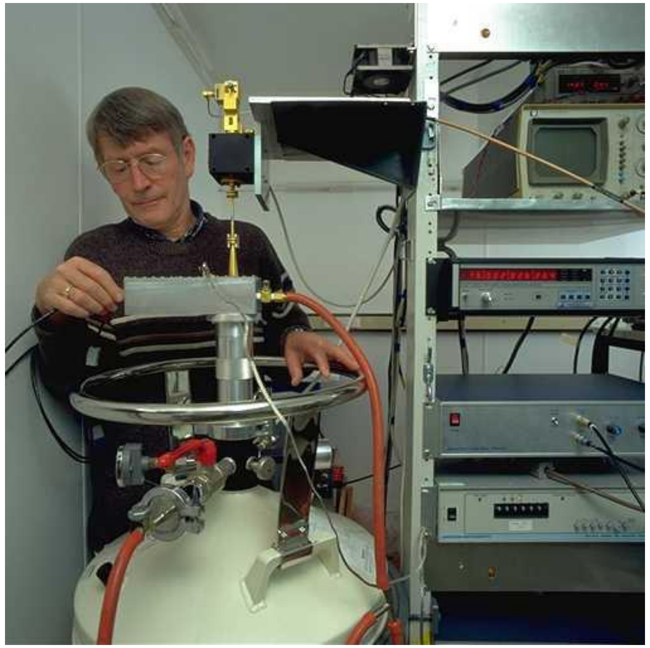
$$\left(\frac{h}{2e}\right) f = 0.14\, mV$$

(for f = 70 GHz, step n = 1)





10 V SIS array



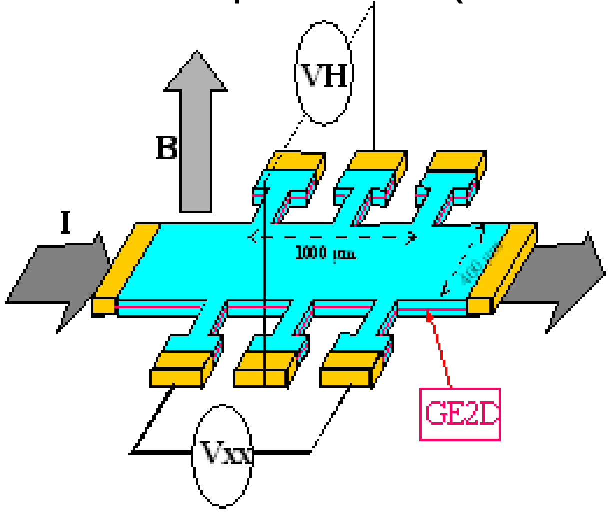
Brian Josephson (1973)

"for his theoretical predictions of the properties of a supercurrent through a tunnel barrier, in particular those phenomena which are generally known as the Josephson effects"

The Quantum Hall Effect

$$\frac{h}{e^2} = 25.8\ k\Omega$$

- Quantum Hall Resistor:
  - 2D electron layer
  - Large magnetic field (~10 T)
  - Low temperatures (< 2.3 K)



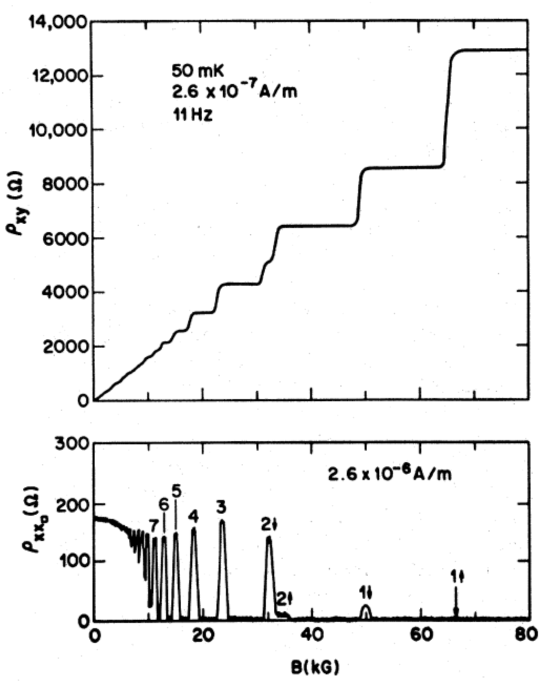
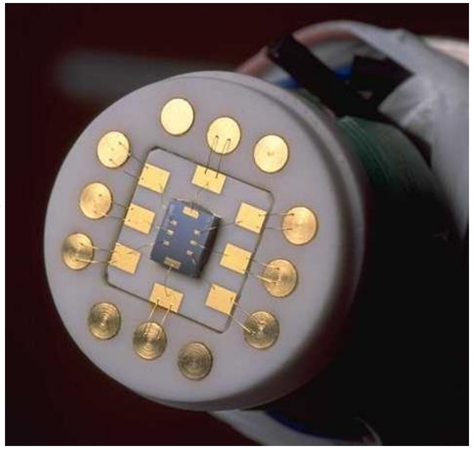
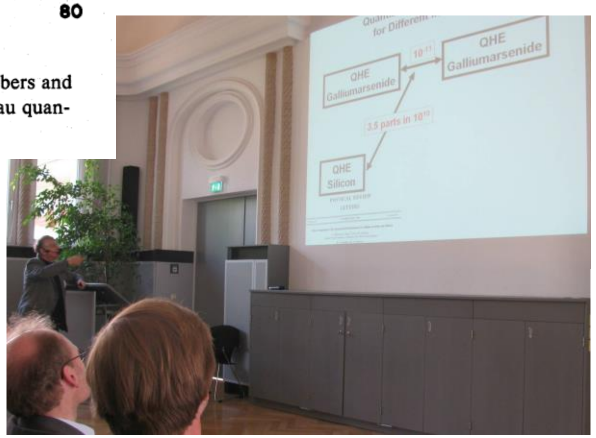


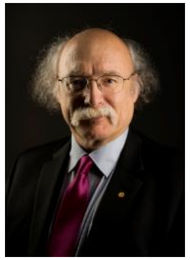
FIG. 1. ρ_{xx} and ρ_{xy} as a function of B. The numbers and the arrows above the ρ_{xx} maxima refer to the Landau quantum number and the spin polarization of the levels.





Directly compare Graphene and GaAs devices: (Janssen *et al*, NPL)

$(R_{GaAs/AlGaAs} - R_{Graphene}) / (h/2e^2) = (-4.7 \pm 8.6) \times 10^{-11}$



Thouless, Haldane, Kosterlitz (2016)

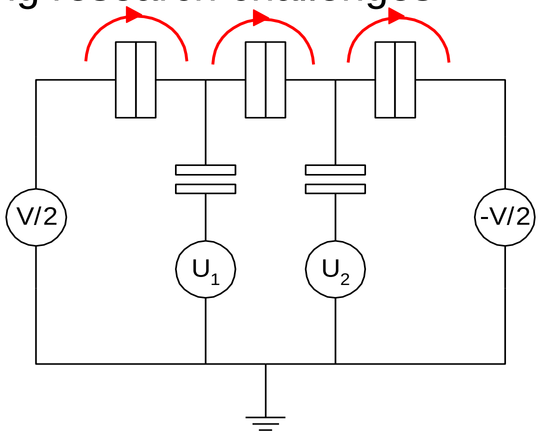
"for theoretical discoveries of topological phase transitions and topological phases of matter"

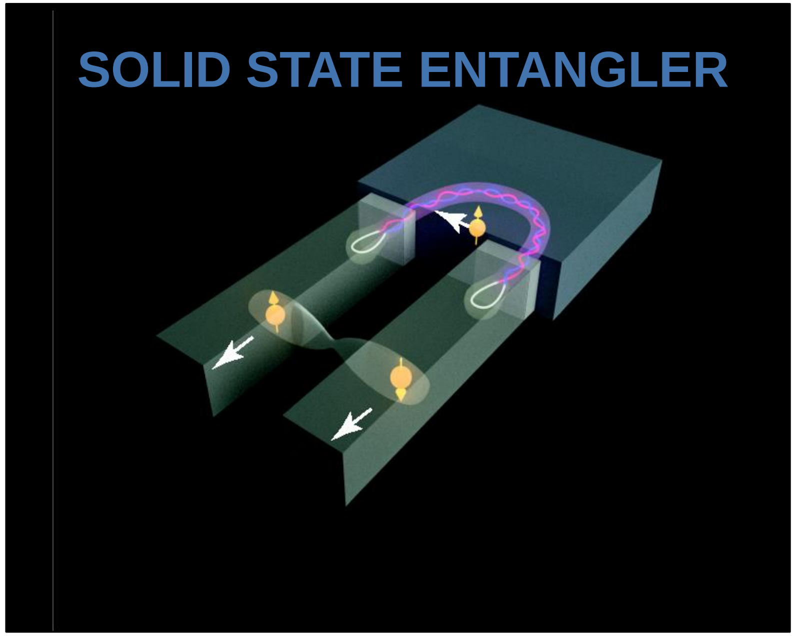
Quantum Current Source

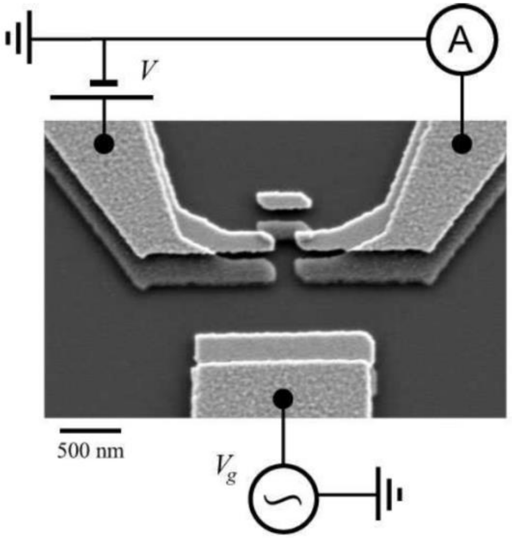
$ef = 1.6\ pA$

(for f = 10 MHz)

- Clocking electrons one by one
  - Currents still too small for high accuracy - ongoing research challenges
- Various device technologies
  - SET (single electron tunneling) turnstile
  - SAW (surface acoustic wave)
  - Tunable barrier pumps, carbon nanotubes etc

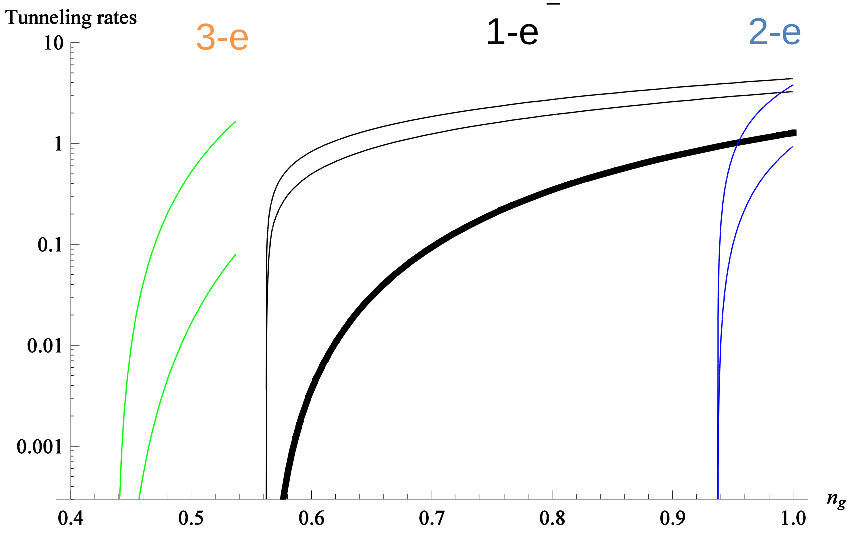






Hybrid superconductor-normal-metal turnstile
Pekola *et al*, Nature Physics 4 (2008) 120

Tunnelling rate vs gate voltage (impedance suppresses unwanted processes)



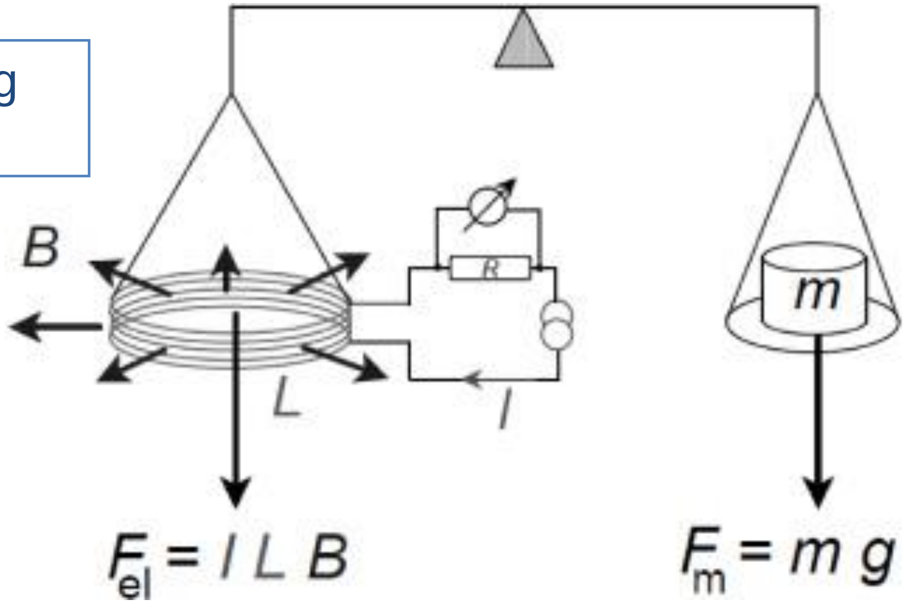
These technologies have strong relevance to:

- future electronic devices (e.g. memory)
- new physics (e.g. qubits for quantum computing)
- manipulation of entanglement – fully quantum

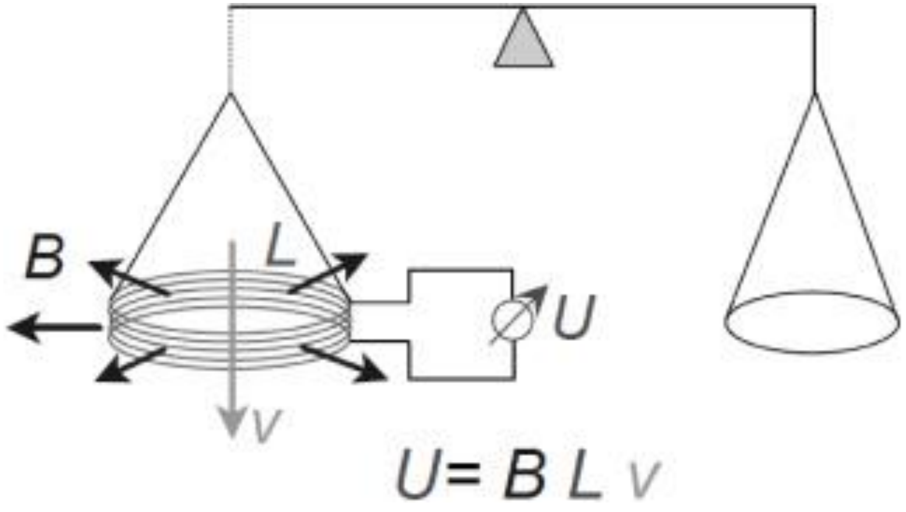
Linking Electrical Units to the SI

- Mechanical versus electrical force (power)
  - Kelvin Current Balance (force generated via mutual inductance of two coils)
  - Kibble: two modes: weighing (static) and calibration (dynamic) – eliminates geometry of magnetic field and coil
  - Weighing current *I*, induced voltage U, coil velocity v

Weighing mode



Calibration mode



Images from: Watt balance experiments for the determination of the Planck constant and the redefinition of the kilogram, M Stock 2013 Metrologia 50 R1

$$\gamma = \frac{m\,g}{I} = \frac{U}{v}$$ so $$m = \frac{U\,I}{g\,v}$$

(these all known to < 10^{-8})

The Kibble Balance

$$m = \frac{U\,I}{g\,v}$$

Equivalence of Electrical and Mechanical Power: $UI = P_{elec} \equiv P_{mech} = mg\,v$

Virtual electrical power: $P_{elec} \sim \frac{U^2}{R}$ since: $I = \frac{U_s}{R}$

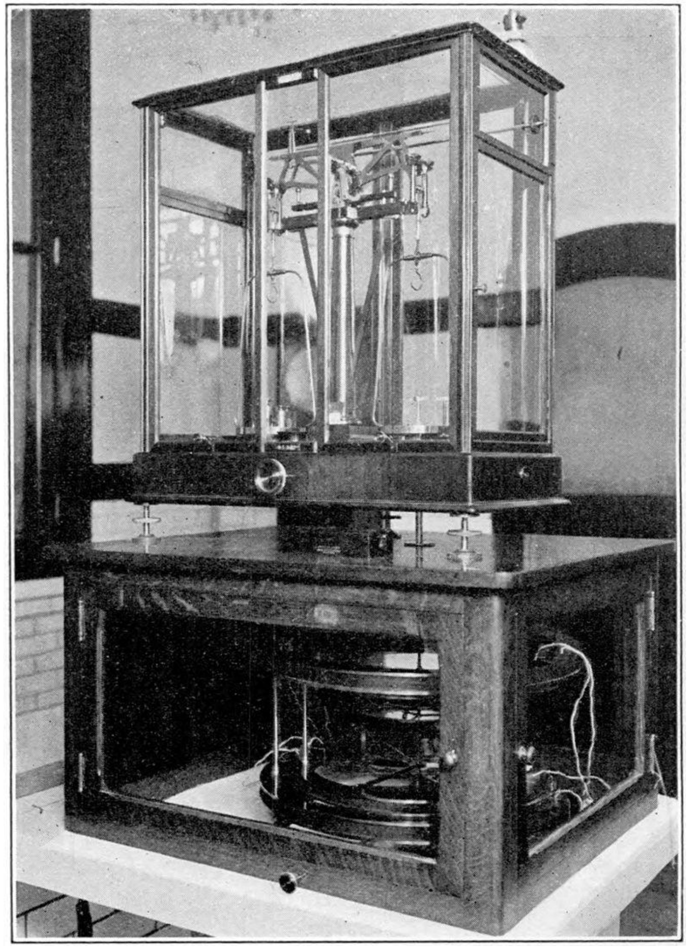
Measure resistance in terms of quantum Hall effect: $R \sim \frac{h}{e^2}$

Measure voltages in terms of Josephson effect: $U \sim \frac{h}{2e} f$

Then the electrical power is related to Planck's constant:
(actually a measure of the Planck constant *h*)

$$P_{elec} \sim \frac{\left(h/2e\right)^2}{h/e^2} \sim h$$

Post Redefinition: define *h* and derive mass scale



MSL Kibble Balance Project – next talk!

The NRC Kibble Balance

- NRC 2017: h = 6.626 070 133 (60) Js
  (standard relative uncertainty: 9.1 x 10^{-9})

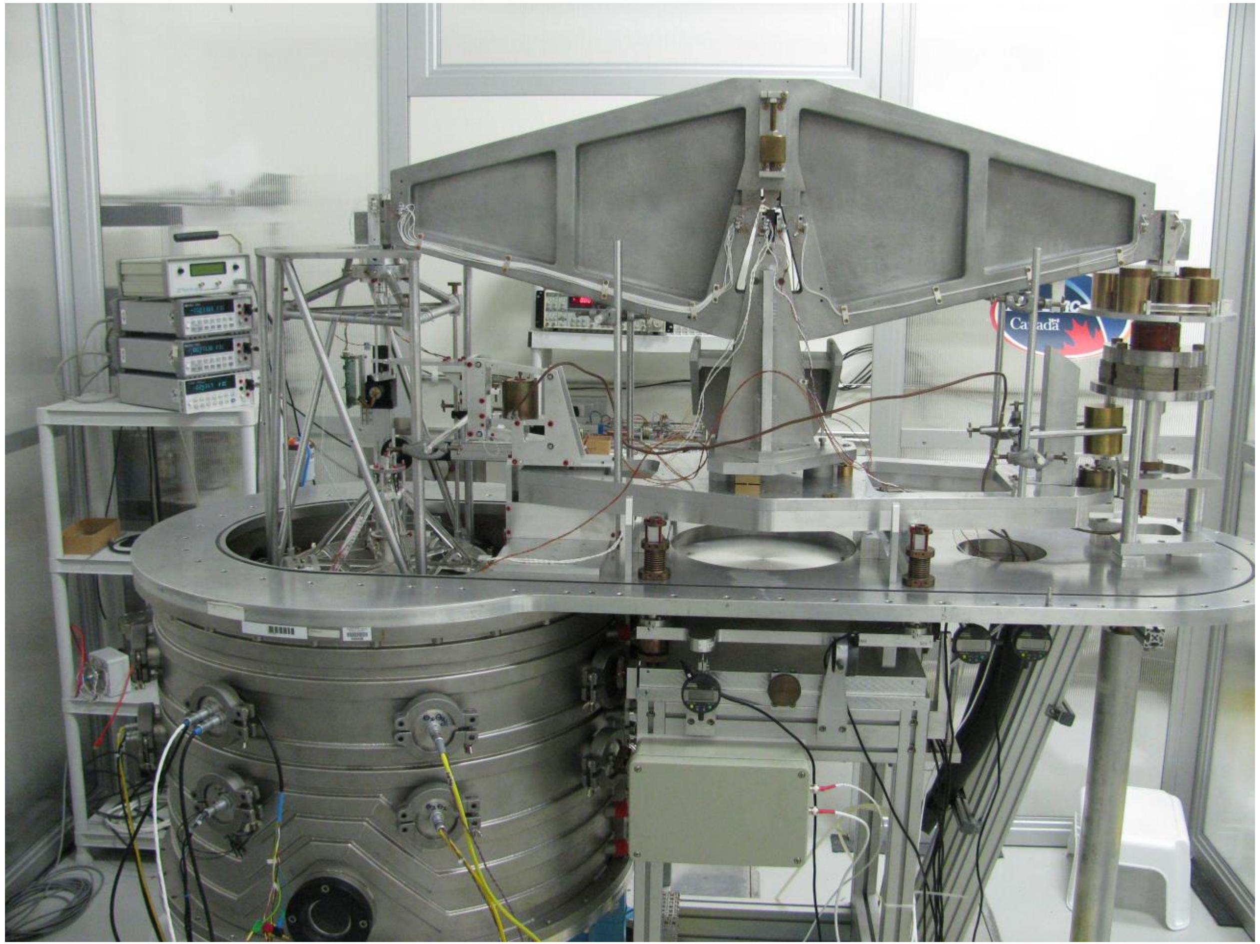


Table A1. The uncertainty budget of the 6th Planck constant determination performed in november 2016 with a 1 kg AuCu mass. The uncertainty index, k, is listed in the first column. The kth uncertainty components are listed in the fourth column are in $u/h_{90} \times 10^9$ and represent $u_{k,6}$. The last seven columns represent the cross correlation vector $CC_{k,6}$.

#	Category	Description	Uncertainty $u/h_{90} \times 10^9$	Correlation with mass#6 determination						
				1	2	3	4	5	6	7
1	Type A	Type A	1.8	0	0	0	0	0	1	0
2	Alignment	$F_x u_x / F_z u_z$	0.08	0	0	0	0	0	1	0
3	Alignment	$F_y u_y / F_z u_z$	0.05	0	0	0	0	0	1	0
4	Alignment	$\tau_x \omega_x / F_z u_z$	0.00	0	0	0	0	0	1	0
5	Alignment	$\tau_y \omega_y / F_z u_z$	0.00	0	0	0	0	0	1	0
6	Alignment	Abbe error correction	0.31	0	0	0	0	0	1	0
7	Alignment	Mass pan alignment	1.46	0	0	0	0	0	1	0
8	Alignment	Laser vertical alignment	0.80	0	0	0	0	0	1	0
9	Alignment	Laser vertical vacuum tilt	1.63	1	1	1	1	1	1	1
10	Alignment	Horizontal displacements	5.31	0	0	0	0	1	1	1
11	Alignment	Vertical displacement	2.80	1	1	1	1	1	1	1
12	Alignment	Changes in θ_z	1.52	1	1	1	1	1	1	1
13	Alignment	Changes in θ_x and θ_y	1.15	1	1	1	1	1	1	1
14	Voltage	Frequency of microwave source	0.21	1	1	1	1	1	1	1
15	Voltage	Filter leakage resistance	0.00	1	1	1	1	1	1	1
16	Voltage	Nano voltmeter gain stability	0.26	0	0	0	0	0	1	0
17	Voltage	Voltmeter non-linearity	0.40	0	0	0	0	0	1	0
18	Voltage	Correlated voltage components	0.40	0	0	0	0	0	1	0
19	Resistance	Measurement versus QHR	3.8	0	0	0	0	0	1	0
20	Resistance	QHR sample dissipation	0.25	0	0	0	0	0	1	0
21	Resistance	Resistor stability	2.6	0	0	0	0	0	1	0
22	Resistance	Resistor power coefficient (1 kg)	0.0	0	0	0	0	0	1	0
23	Velocity	Laser calibration	0.03	0	0	0	0	0	1	1
24	Velocity	Mode leakage	0.01	1	1	1	1	1	1	1
25	Velocity	Diffraction correction	0.5	1	1	1	1	1	1	1
26	Velocity	Retro reflector imperfections	0.20	1	1	1	1	1	1	1
27	Velocity	Beam shear	0.30	1	1	1	1	1	1	1
28	Velocity	Frequency measurement	0.13	0	0	0	0	0	1	0
29	Velocity	Position measurement	0.90	1	1	1	1	1	1	1
30	Velocity	Index of refraction	0.00	0	0	0	0	0	1	0
31	Velocity	Modeling uncertainty	1.00	1	1	1	1	1	1	1
32	Velocity	Velocity dependence	2.60	1	1	1	1	1	1	1
33	Velocity	Trigger delay	0.06	1	1	1	1	1	1	1
34	Mass	Traceability to IPK	3.5	1	1	1	1	1	1	1
35	Mass	Prototype drift model	1.0	0	0	0	0	1	1	1
36	Mass	Balance uncertainties	2.0	0	0	0	0	0	1	0
37	Mass	Vacuum calibration cycling stability	0.0	0	0	0	0	0	1	0
38	Mass	Watt balance transfer mass stability	1.5	0	0	0	0	0	1	0
39	Mass	Pressure dependence	0.1	0	0	0	0	0	1	0
40	Mass corr.	Sorption on reference mass	1.5	0	0	0	0	0	1	0
41	Mass corr.	Center of gravity	0.1	1	0	0	0	0	1	0
42	Mass corr.	Weighing range sensitivity	0.1	0	0	0	0	0	1	0
43	Gravity	Absolute gravity measurement	2.7	0	0	0	0	0	1	1
44	Gravity	Horizontal transfer	2.5	1	1	1	1	1	1	1
45	Gravity	Vertical transfer	3.0	1	1	1	1	1	1	1
46	Grav. corr.	Balance attractions	1.00	1	1	1	1	1	1	1
47	Grav. corr.	Earth tides	0.01	1	1	1	1	1	1	1
48	Grav. corr.	Polar motion	0.10	1	1	1	1	1	1	1
49	Grav. corr.	Ocean loading	0.29	1	1	1	1	1	1	1
50	Grav. corr.	Atmospheric pressure	0.25	1	1	1	1	1	1	1
51	Gravity	Site dependence	1.0	0	0	0	0	0	1	1
52	Weighing	Mass exchange errors	1.70	1	1	1	1	1	1	1
53	Weighing	Knife edge hysteresis	0.98	1	1	1	1	1	1	1
54	Weighing	Buoyancy in watt balance	0.00	0	0	0	0	0	1	0
55	Weighing	Magnetization 1st order	0.00	1	1	1	1	1	1	1
56	Weighing	Magnetization 2nd order	5.26	1	1	1	1	1	1	1
57	Weighing	Mag. hysteresis correction uncertainty	0.37	0	0	0	0	0	1	0
58	Weighing	Susceptibility	0.06	1	0	1	0	0	1	1

Link between N_A and h

- alternative route to h through silicon sphere
- Isotopically purified (Si-28)
    - cost ~ €1M each? (PTB getting 5)
- Measure lattice spacing and density (XRCD)

Link between N_A and h:

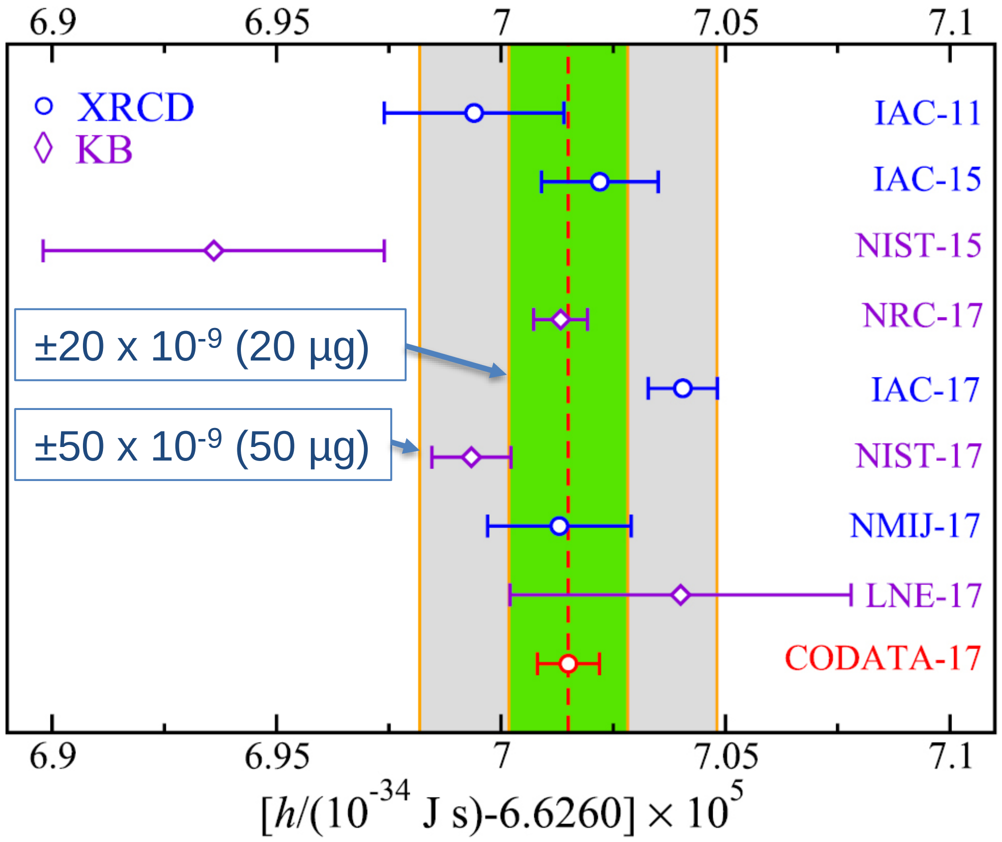
$$N_A = \frac{A_r(e) M_u c \alpha^2}{2 R_\infty h}$$

$M_u = 1 \frac{\text{g}}{\text{mol}}$ (exact)

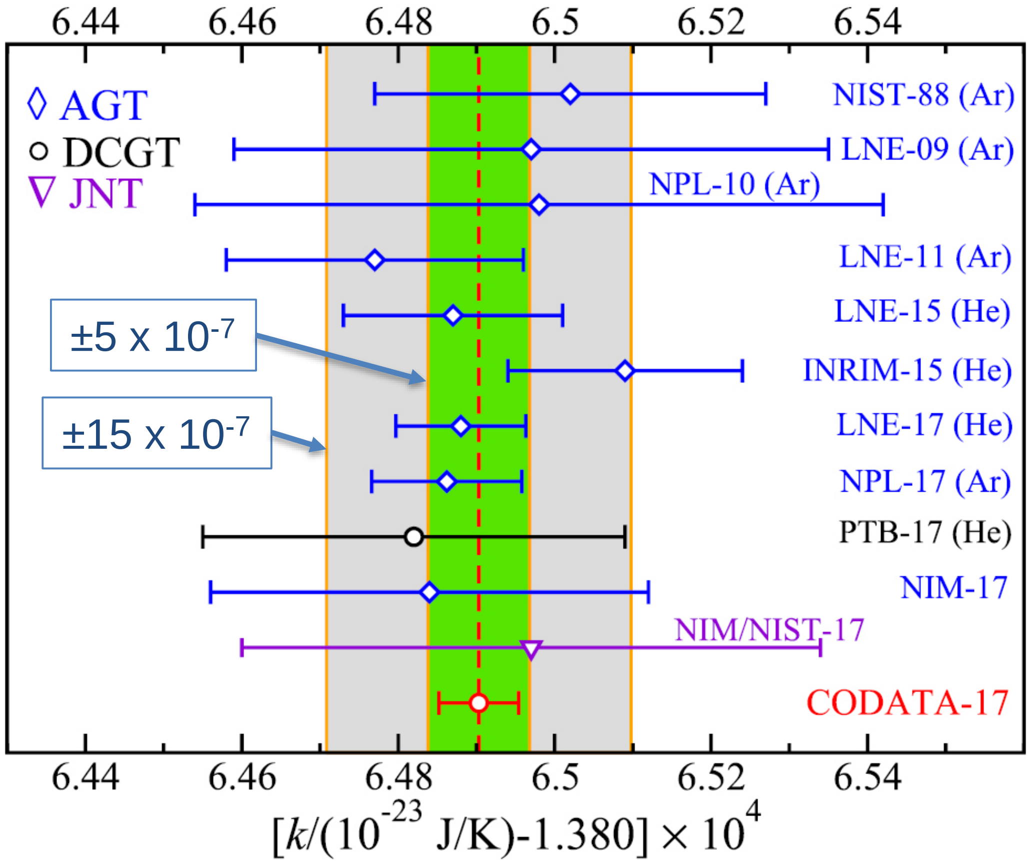
R_∞ = Rydberg constant

$A_r(e)$ = relative atomic mass of the electron

CODATA 2017 Results



Planck constant results (in chronological order)
(KB: Kibble balance; XRCD: x-ray-crystal-density)



Boltzmann constant results (in chronological order)
(AGT: acoustic gas thermometry; DCGT: dielectric constant gas thermometry; JNT: Johnson noise thermometry)

[from: The CODATA 2017 values of h, e, k, and NA for the revision of the SI
D B Newell et al 2018 Metrologia 55 L13]

Proposed Changes to the SI

- To be based on 7 fixed constants:

	To be fixed in 2019		Already fixed
1	Planck's constant (the central constant of quantum mechanics) h = 6.626 070 15 ×10^{-34} J s	5	Speed of light (the central constant of relativity) c = 299 792 548 m s^{-1} (1983)
2	The elementary charge (the central constant of matter) e = 1.602 176 634 ×10^{-19} C	6	Hyperfine splitting of Caesium 133 $\Delta\nu_{Cs}$ = 9 192 631 770 Hz (1967)
3	Boltzmann's constant ($E_{thermal}$~kT) k = 1.380 649 ×10^{-23} J K^{-1}	7	Luminous efficacy of 540 THz radiation K_{cd} = 683 lm W^{-1} (1979)
4	Avogadro's constant N_A = 6.022 140 76 ×10^{23} mol^{-1}		

"The revision of the SI will ensure that the SI continues to meet the needs of science, technology, and commerce in the 21st century." (BIPM)

- changes small but conceptually very different basis

Table 2. The CODATA 2017 adjusted values of h, e, k, and N_A.

Quantity	Value	Rel. stand. uncert u_r
h	$6.626\,070\,150(69) \times 10^{-34}$ J s	1.0×10^{-8}
e	$1.602\,176\,6341(83) \times 10^{-19}$ C	5.2×10^{-9}
k	$1.380\,649\,03(51) \times 10^{-23}$ J K^{-1}	3.7×10^{-7}
N_A	$6.022\,140\,758(62) \times 10^{23}$ mol^{-1}	1.0×10^{-8}

Table 3. The CODATA 2017 values of h, e, k, and N_A for the revision of the SI.

Quantity	Value
h	$6.626\,070\,15 \times 10^{-34}$ J s
e	$1.602\,176\,634 \times 10^{-19}$ C
k	$1.380\,649 \times 10^{-23}$ J K^{-1}
N_A	$6.022\,140\,76 \times 10^{23}$ mol^{-1}

Proposed Electrical Changes to the SI

- K_J will change by ~0.1 ppm –will affect voltage references slightly $K_J = \frac{2e}{h}$

- R_K will change by ~0.02 ppm – only affect QHR realisations? $R_K = \frac{h}{e^2}$

- μ_0, ε_0, and Z_0 will become experimentally determined $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$
    - $\mu_0 = 4\pi\ [1 + 0.0(2.3) \times 10^{-10}] \times 10^{-7}$ N A^{-2}
    - uncertainty: $u_r(\mu_0) = u_r(\alpha)$ since: $\mu_0 = \alpha \frac{2h}{ce^2}$

- voltage and resistance uncertainties < 0.1 ppm will be fully traceable to the SI

SI Revision: Timeline

- July 1, 2017: Deadline of acceptance of new data. ✓
- Sept 4, 2017: CODATA TGFC meeting, manuscripts must be already accepted and publicly available. ✓
- Sept 5-6, 2017: CCU reviews values recommends digits etc. ✓
- Oct 16-20, 2017: CIPM meeting – recommendation to the CGPM. ✓
- Nov 13-24, 2018: CGPM approves the ‘Revised SI’ (*fait accompli?*).
- May 20, 2019: Implementation day.

- WMD 2018 to WMD 2019: one year period for promotion (international)
- MSL will be hosting WMD events (like this) and supporting the change
- Possibility for new data during this time….?

Responsibilities

- CGPM makes the decision (diplomatic level)
- Consultative Committees of the BIPM are responsible for operational aspects
  (MSL represented on CCEM, CCM, CCPR and CCT)
- Draft *Mise en Pratique (practical implementation)* for kg, A, mol, K (for post definition)
  - Evolve with technology changes without affecting definition
  - These are the techniques used by MSL and other NMIs to realise the definitions

Mise en pratique of the definition of the kilogram

Consultative Committee for Mass and Related Quantities (CCM)
Working Group on the Realization of the Kilogram (WGR-kg)

(*Editor's note 0.1*: In the following text, all digits in red are meant to be place holders for the final digits to be inserted at the time of approval of the redefinition.)

1. Introduction

1.1 Definition of the kilogram

***Mise en pratique* for the ampere and other electric units in the International System of Units (SI)**

CCEM Working Group on the SI
Draft #1

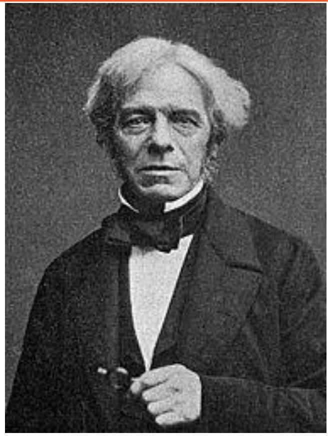
1. Introduction. Resolution AA of the 24th General Conference on Weights and Measures (CGPM), which convened in October 2011, abrogated the then existing definitions of the four base units kilogram, ampere, kelvin, and mole of the International System of Units (SI) and replaced them with new definitions that fix the values of the Planck constant h, elementary charge e, Boltzmann constant k, and Avogadro constant N_A, respectively. The exact values of h,

Resources – see www.bipm.org

- 8th edition of SI Brochure (2014): http://www.bipm.org/en/publications/si-brochure/
- Draft 9th edition: http://www.bipm.org/utils/common/pdf/si-brochure-draft-2016b.pdf
- Brandbook: http://www.bipm.org/utils/common/pdf/SI-Brand-Book.pdf
- Website (articles, videos etc): http://www.bipm.org/en/measurement-units/rev-si/

The Particular Challenge of Electrical Units

- The discovery of electromagnetism (Faraday 1812, Ørsted 1820)
  - Multiple units systems e.g. Electrostatic-CGS, Electromagnetic-CGS, Gaussian, Heaviside…
  - Freedom in choice of unit, numbers and constants in equations (e.g. 4π)
  - Controversy involving some great names (Gauss, Weber, Maxwell, Heaviside, Lorentz, ….)
- CGS has certain advantages – some prefer to teach electromagnetism in CGS
  - 1.5 volt = 0.005 statvolt (ESU) = 1.5×10^8 abvolt (EMU)
  - 1 ampere = 3×10^9 statampere (ESU) = 0.1 abampere (EMU)
- Consistent use of SI works well – convenient measures but conceptual difficulties



Michael Faraday

James Clerk Maxwell

“Scientists have spent almost a century disagreeing about the units for electromagnetism” (J H Williams)

Electrical Units – the Ampere

- Force laws – defines link to mechanical quantities (also via energy or power)
  - Two choices:

    Coulomb's Law: $F_e = k_e \frac{q_1 q_2}{r^2}$ *Ampère's Force Law:* $\frac{dF_m}{dl} = 2k_m \frac{I_1 I_2}{d}$

    linked by: $I = \frac{dq}{dt}$ requiring: $\frac{k_e}{k_m} = c^2$

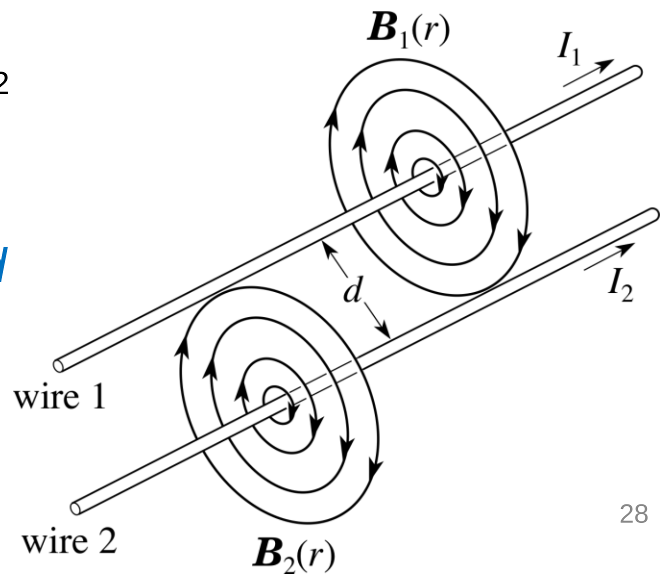
    (free to define k_e or k_m but not both)

d = 1 m
$I_1 = I_2$ = 1 ampere
dF/dl = 2x10^{-7} N/m

- SI Definition of the ampere employs Ampère's force law with $k_{mSI} = 10^{-7}$ N/A^2

  "The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed one metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newtons per metre of length."

- Why $k_{mSI} = 10^{-7}$ N/A^2 ?



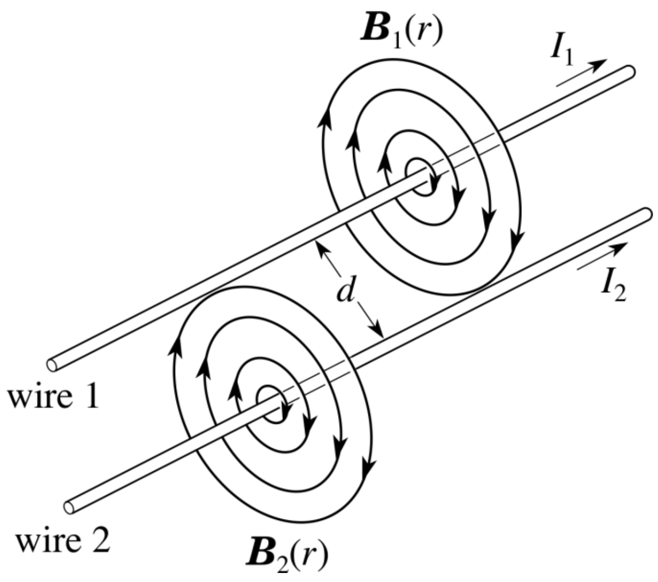
Electrical Units – CGS

d_{CGS} = 1 cm
I_{1CGS} = I_{2CGS} = 1 abampere
dF_{mCGS}/dl = 2 dyne/cm

- Using CGS-EMU for Ampère's Law:

$$k_{mCGS} = 1 \qquad \frac{dF_{mCGS}}{dl} = 2\frac{I_{1CGS}I_{2CGS}}{d_{CGS}}$$

- For I_{1CGS} = I_{2CGS} = 1 abampere, d_{CGS} = 1 cm then dF_{mCGS}/dl = 2 dyne/cm
- Not really a base unit (depends on force)
- Maxwell: *"Every electromagnetic quantity may be defined with reference to the fundamental units of Length, Mass and Time"*
  - In CGS dimensions of I are: $g^{½}$ $cm^{½}$ s^{-1} (messy)
- For MKSA Giorgi's proposal: derive ampere from force law then call it a base unit (simplifies)



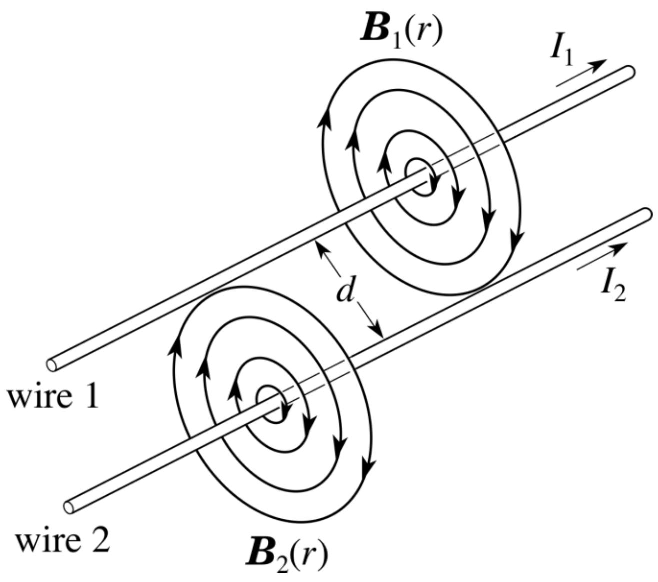
Electrical Units – Convert CGS Definition to SI

d_{SI} = 10^{-2} m
$I_{1SI} = I_{2SI}$ = 10 ampere
dF_{mCGS}/dl = 2x10^{-3} N/m

- dF_{mCGS}/dl = 2 dyne/cm
  - convert to SI: (1 dyne = 10^{-5} N, 1 cm = 10^{-2} m) so dF_{mCGS}/dl = 2 x10^{-3} N/m
- Express quantities in SI units: $2 \times 10^{-3}\text{N/m} = 2k_{mSI}\dfrac{I_{1SI}I_{2SI}}{d_{SI}}$
- Since $I_{1SI} = I_{2SI}$ = 10 ampere (using 1 abampere = 10 A), d_{SI} = 10^{-2} m,

  then $2k_{mSI}$ = 2 x10^{-7} N/A^2
- To avoid factors of π in field relationships, write: $2k_m = \dfrac{\mu_0}{2\pi}$

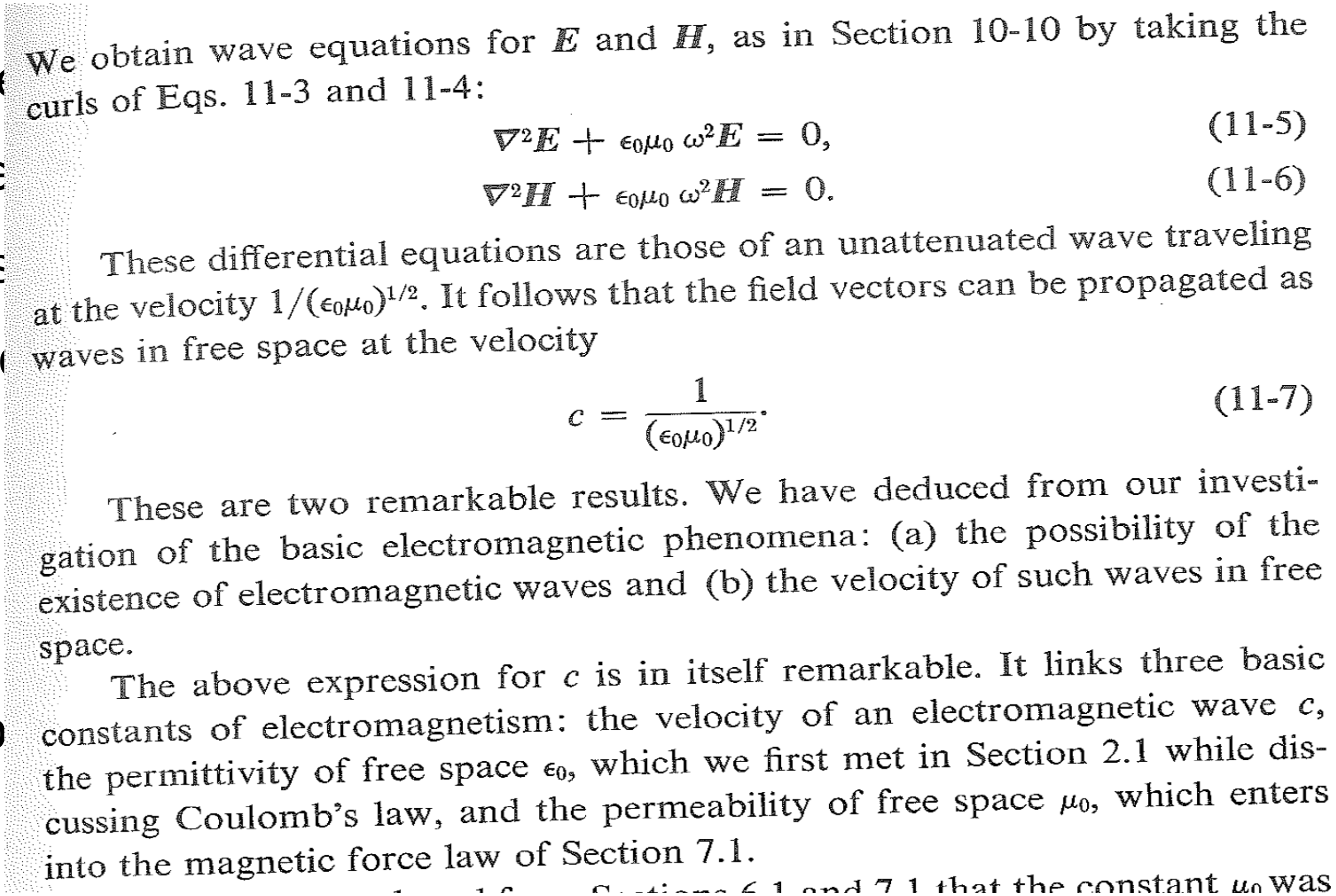
  (called “rationalisation”)
- Then: μ_0 = 4π x10^{-7} N/A^2 (the magnetic constant, conversion factor for the SI)
- Similarly the electric constant for Coulomb’s Law: $k_e = \dfrac{1}{4\pi\varepsilon_0}$ so $\dfrac{1}{\mu_0\varepsilon_0} = c^2$ and $\sqrt{\dfrac{\mu_0}{\varepsilon_0}} = Z_0$



Curious about ε_0 and μ_0?

- μ_0 = 4π x10^{-7} N/A^2 is called the 'perme
- ε_0 = 8.854187 x10^{-12} F/m is called the
- ε_0 and μ_0 do not appear in other unit s
- Just conversion factors for the SI – n
- Misinterpretation of $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$ (e.g

  (c does not depend on ε_0 and μ_0)
- What about the impedance of free sp
- Trend to call μ_0 the 'magnetic constant' and ε_0 the 'electric constant'
- Redefinition: e_{SI} is fixed but not e_{CGS} !



We obtain wave equations for E and H, as in Section 10-10 by taking the curls of Eqs. 11-3 and 11-4:

$$\nabla^2 E + \epsilon_0\mu_0\,\omega^2 E = 0, \qquad (11\text{-}5)$$

$$\nabla^2 H + \epsilon_0\mu_0\,\omega^2 H = 0. \qquad (11\text{-}6)$$

These differential equations are those of an unattenuated wave traveling at the velocity $1/(\epsilon_0\mu_0)^{1/2}$. It follows that the field vectors can be propagated as waves in free space at the velocity

$$c = \frac{1}{(\epsilon_0\mu_0)^{1/2}}. \qquad (11\text{-}7)$$

These are two remarkable results. We have deduced from our investigation of the basic electromagnetic phenomena: (a) the possibility of the existence of electromagnetic waves and (b) the velocity of such waves in free space.

The above expression for c is in itself remarkable. It links three basic constants of electromagnetism: the velocity of an electromagnetic wave c, the permittivity of free space ϵ_0, which we first met in Section 2.1 while discussing Coulomb's law, and the permeability of free space μ_0, which enters into the magnetic force law of Section 7.1.

... Sections 6.1 and 7.1 that the constant μ_0 was

The SI Conceptual Difficulties

- The role of μ_0 and ε_0 – these do not exist in CGS
- The nature of various fields:
  - SI: B and H have different dimension (not so in CGS)
  - SI: E and D have different dimension (not so in CGS)
  - SI: B and E have different dimension (not so in CGS)
  - SI: $E/H = Z_0$ (not so in CGS)
  - CGS: all fields have same dimension (some different units)

	SI	CGS
Electric Fields in Materials	$D = \varepsilon_r \varepsilon_0 E$	$D = \varepsilon_r E$
Magnetic Fields in materials	$B = \mu_r \mu_0 H$	$B = \mu_r H$
Lorentz Force	$F = q(E + v \times B)$	$F = q\left(E + \frac{v}{c} \times B\right)$

On the Establishment of Fundamental and Derived Units, with Special Reference to Electric Units. Part II

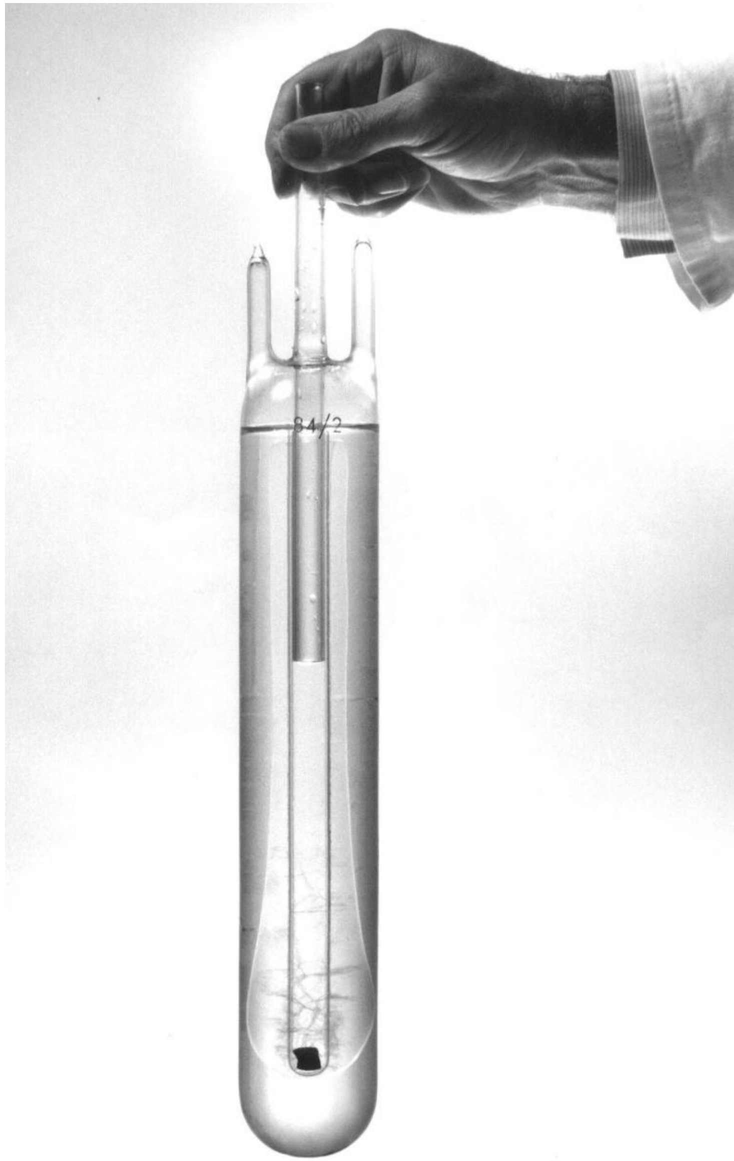
R T Birge, Am Phys Teacher 3, p171, 1935

"The discussions of the IEC centred on the question as to whether the physical magnitudes B and H were of the same character or not. In spite of H. Abrahams' clear and convincing reports to the contrary, the IEC decided that B and H had different physical characters. I do not agree with this decision….

Meanwhile one can hope that the point of view of the IEC will not be accepted by physical scientists."

Consequences

- The last artefact, the IPK, will have been replaced.
  - Need for practical mass realisations derived from electrical quantities.
- Electrical measurements fully covered by the SI.
  - μ_0 or ε_0 will need to be determined experimentally
  - Exact relationship between SI and CGS broken (for electrical quantities)
  - Should avoid CGS magnetic units (Gauss etc)
- The triple point of water will become experimentally determined.
  - Previously the thermodynamic temperature of the triple point of water was defined as exactly 273.16 kelvin
  - CCT *Mise en pratique* will suport use of ITS-90 for now
- Mole will no longer link to mass (role of ^{12}C ends).



Summary – is this the last SI revolution?

- Fundamental constant definitions unlikely to be usurped
    - As anticipated by Maxwell, Planck…
- Much better atomic clocks being developed (e.g. optical clocks)
    - A fundamental constant for the second?
- New discoveries – a new quantum effects?
- New physical theories (e.g. could reduce number of independent physical constants)
- Significant technology changes…
- But should be good for the 21st century (first half anyway…)

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